## Frustration

Contra Leibniz, the solution to the problem of evil - if this is the best if all possible worlds, then why can't I smoke cigarettes? - does not require an appeal to unfathomably infinite complexity or the inscrutable wisdom of God, but rests on a relative triviality, the phenomenon called frustration in statistical mechanics. Essentially in all but the simplest cases the desired optimum is either ill-defined or does not exist.

That is, the behavior of dissipative physical systems can be described teleologically, as a tendency toward attaining the state of lowest energy.

A simple model of matter in the solid state illustrating the principle might consist of a regular lattice, with "atoms" of some kind located at the vertices, adjacency indicated by an edge connecting two vertices, and energies computed by summing energies of interaction between nearest neighbors which depend upon both their states.

So you might, for instance, try to find the state of lowest energy for a planar network of spins with values $\pm 1$, where the contribution of each edge to the total interaction energy is the product of the values at its endpoints. - You can think of the spins as little bar magnets, for instance, which when adjacent will attempt to align their poles north-to-south. - Then it is easy to see that if the number of edges around any closed circuit is even, as with a square lattice, then all the spins can be oriented oppositely to their neighbors, every edge contributes a net of -1 to the total energy, and there are globally two symmetric but distinct solutions of minimum energy, one flipping all the spins of the other.


If on the other hand there are circuits with an odd number of edges, as with a triangular lattice, there is no way to satisfy the constraints to produce a stable ${ }^{1}$ ground state; thus in this model, practically the simplest in which the question makes sense, there is no unique best of all possible worlds.


[^0]Obviously in more complicated situations the problem only gets worse, and it is conjectured, for instance, that the peculiarly amorphous structure of glasses is the consequence of the mutual microscopic interactions of the constituents being sufficiently complex that either an unambiguous ground state is nonexistent, or that for the system to find the solution of the equivalent optimization problem would represent a computation which could not be completed within, say, the age of the universe.

So if you were to insist on interpreting the valuation in moralistic terms - if you were to say, for instance, that magnets can only be happy when they are aligned south to north and north to south with each of their neighbors - then that is the origin of evil: in a world of more than minimal complexity, it is mathematically impossible to avoid it. ${ }^{2}$

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The same picture can be applied to the problem of synchronicity: suppose the universe consists of an infinite planar triangular lattice:

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in which again there are spins at every vertex, flipped up or down, and they interact along edges, with a contribution to the energy which is positive if they point in the same direction and negative if they are oppositely aligned.

Then for any triangle there are essentially only two distinct configurations, [1] two spins the same and one different (net energy -1 ), and [2] all spins the same (net energy +3 ):


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\epsilon=+3
$$



In general, then, how must the spins be arranged?
I won't pretend this argument is rigorous, but rephrase the rules as follows: begin with the triangular lattice, and two-color the edges without forming any black triangles; red edges correspond to pairs of vertices with the same spin, counting +1 , and black edges to pairs with opposite spin, counting -1 . Then minimal energy will be attained when a maximal number of black edges are colored in, and this occurs when the lattice has been tiled with tiles composed of two triangles with two black and one red edge apiece, meeting on the red edge:

A collection of such tiles with v vertices will have energy $(1-\mathrm{v})$. Thus for the arrangement of seven hexagons as follows, with 31 vertices and 72 edges, a minimum configuration colors 51 edges black and 21 red, with energy -30 :

These tilings are not guaranteed to be periodic, but if the additional rule/constraint is added that the black edges tile the plane in hexagons:
then the arrangement has the property that the spins at the centers are arbitrary, and can be up or down as you please:



- i.e. the degeneracy is (uncountably) infinite. And if the system were to assume this configuration and were subject to thermal agitation, the central spins would flip up and down randomly ad infinitum.

But if we remove thermal agitation and freeze the state, what else might that mean? That you can code anything you like into it. Out of the set of possible worlds satisfying these laws of physics, you can select one in which the hexagonal cells are black/white pixels, and it's a picture of Mickey Mouse, or a cartoon, or a frame by frame display of Chopper Chicks in Zombietown. ${ }^{3}$ Or groups of eight hexagons are bytes corresponding to the ASCII character set, and the vacuum is spelling out The Wind in the Willows, or Finnegan's Wake, or the whole of Borges' Library of Babel at once. Or they're a representation of musical notation, and the vacuum is humming the Goldberg Variations. Such a world could have an infinite variety of meanings superimposed upon it.

Better yet, call the rules for computing the energy of the vacuum Physics-1, take a minimum, and leave the internal spins of the

[^2]hexagons arbitrary. Now draw a second series of edges from the centers of the hexagons to one another:

and define a second energy, exactly like the first, in which the hexagons become the new vertices with spins defined by the ambiguous internal values. Call this Physics-2. You can then, without affecting any of the results of Physics-1, try to minimize the energy of this second-order vacuum according to the same rules, the solution will take the same form, and there will be a residual ambiguity which you can attempt to resolve by rescaling again and imposing Physics-3.

- And so on.

Thus even in this simple model it is possible to imagine an infinite hierarchy of independent physical worlds built up on one another, all referring to the same primitive data. So that even if the principle of sufficient reason ultimately applied, and there were a reason determining the value of every spin in the lattice, the theory that
would explain it all would have to have infinite complexity. ${ }^{4}$ - Worse than that, at every level of explanation, for every Physics-n, it would be easy to fool yourself into thinking you had said everything there was to say about the structure of the world, since every regularity you had overlooked would have been swept under the rug of "initial conditions".

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[^0]:    ${ }^{1}$ The physical picture here is that there is always some inherent jiggle in the system - heat, measured by a temperature - that agitates the magnets randomly and continuously, allowing them to change their state even if it means temporarily adopting a configuration of higher energy. In the case of a triangle in the state ++ , this could be a transition to +++ and then to any of,,,,+-+-+++--+--+ . Thus a degenerate ground state at finite temperature is inherently unstable.

[^1]:    ${ }^{2}$ I suppose this must sound ridiculous, but this is exactly what Leibniz meant, and this is how to solve his problem. - As for why he would not have thought of something like this, I imagine it was because his logic excluded relations.

[^2]:    ${ }^{3}$ Written/directed by Dan Hoskins, 1989.

[^3]:    ${ }^{4}$ Note, incidentally, that even the infinite tower of \{Physics-n\} would not determine the values of all the spins: at each level there would be an additional =/- degeneracy from flipping the perimeter spins, and the result would be uncountably degenerate.

